SPLIT-UNIT (split-plot) DESIGN

1 DESCRIPTION

An experiment in which an extra factor (second) is introduced into a study by dividing the large experimental units(whole unit) for the first factor into smaller experimental units(sub-units) on which the different levels of the second factor will be applied. Each whole unit is a complete replicate of all the levels of the second factor (RBD). The whole unit design may be CRD, RCBD or LS design.

Randomization - The first factor levels are randomly assigned to the whole units according to the rules for the whole unit design (i.e., CRD, RCBD or LS design). While the second factor levels are randomly assigned sub-units within each whole unit according to the rules of a RCBD. The name of the split-plot design is prefixed with the design name associated with the whole plot design, i.e., Randomized Complete Block Split-Plot Design. The design for the sub-plot is never given, but is assumed since it must by definition be RBD.

2 ADVANTAGES

2.1 Since sub-unit variance is generally less than whole unit variance, the sub-unit treatment factor and the interaction are generally tested with greater sensitivity.

2.2 Allows for experiments with a factor requiring relatively large amounts of experimental material (whole units) along with a factor requiring relatively little experimental material (sub -unit) in the same experiment.

2.3 If an experiment is designed to study one factor, a second factor may be included at very little cost.

2.4 It is the design (univariate) for experiments involving repeated measures on the same experimental unit (whole unit), while the repeated measures in time are the sub-unit.

3 DISADVANTAGES

3.1 Analysis is complicated by the presence of two experimental error variances, which leads to several different SE for comparisons.

3.2 High variance and few replications of whole plot units frequently leads to poor sensitivity on the whole unit factor.

4 POSSIBLE APPLICATIONS

4.1 Experiments in which one factor requires larger experimental units than the other factor.

4.2 Experiments where greater sensitivity may be desired for one factor than for the second factor.

4.3 Introduction of a new factor into an experiment which is already in progress.

5 Example: RCB split plot, with a 3x2 factorial arrangement of treatments.

Rep I

a_2	a_1	a_3
b2	b2	b1
b1	b1	b2

Rep II

a_3	a_1	a_2
b2	b1	b2
b1	b2	b1

Other reps the same except a new randomization for each rep.

6 The Model RCB Split Plot

6.1 Linear additive model

$$Y_{ijk} = \mu + \rho_k + \alpha_i + \delta_{ik} + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

where: Y_{ijk} is the observed value for the k^{th} replicate of the i^{th} level of factor A and the j^{th} level of factor B (where i = 1 to a, j = 1 to b and k = 1 to r).

μ is the general mean.

ρ_k is the block effect for the k^{th} block; the block effect may be either fixed or random.

α_i is the effect of the i^{th} level of factor A; the effect may be either fixed or random.

δ_{ik} is the whole plot random error effect, for the i^{th} , k^{th} combination of block and factor A.

β_j is the effect for the j^{th} level of factor B; the effect may be either fixed or random.

$\alpha\beta_{ij}$ is the interaction effect of the i^{th} level of factor A with the j^{th} level of factor B; the interaction effect may be either fixed or random

ϵ_{ijk} is the subplot random error effect associated with the Y_{ijk} subplot unit.

7 The Analysis of Variance

7.1 The expected means squares

RCB split-plot design (blocks random), whole unit treatment is factor A, while the sub-unit treatment is factor B:

Sources	df	Expected Mean Squares
Whole plot analysis:		
Blk	r-1	$\sigma_\epsilon^2 + b\sigma_\delta^2 + ab\sigma_{Blk}^2$
A	a-1	$\sigma_\epsilon^2 + b\sigma_\delta^2 + rb\theta_A^2$
Error a	(r-1)(a-1)	$\sigma_\epsilon^2 + b\sigma_\delta^2$
Sub-plot analysis:		
B	b-1	$\sigma_\epsilon^2 + ra\theta_B^2$
A*B	(a-1)(b-1)	$\sigma_\epsilon^2 + r\theta_{AB}^2$
Error b	(r-1)a(b-1)	σ_ϵ^2
Total	rab - 1	

7.2 Standard error of the means for an A main effect mean

$$SEM_A = \sqrt{\frac{S_{\epsilon}^2 + b\left(S_{\delta}^2 + S_{blk}^2\right)}{rb}}$$

for a B main effect mean

$$SEM_B = \sqrt{\frac{S_{\epsilon}^2 + S_{\delta}^2 + aS_{blk}^2}{ra}}$$

for an AB mean

$$SEM_B = \sqrt{\frac{S_{\epsilon}^2 + S_{\delta}^2 + S_{blk}^2}{r}}$$

7.3 Standard errors of the differences between means

$$SED_A = \sqrt{\frac{2\,(S_\epsilon^2 + S_\delta^2)}{rb}}$$

$$SED_B = \sqrt{\frac{2S_\epsilon^2}{ra}}$$

at the same level of A, different levels of B

$$SED_{AB} = \sqrt{\frac{2S_\epsilon^2}{r}}$$

at the same level of B, different or same levels of A

$$SED_{AB} = \sqrt{\frac{2\left(S_\epsilon^2 + \frac{1}{b}S_\delta^2\right)}{r}}$$

7.4 Sources of variation and degrees of freedom for various split-plot designs, with a two factor factorial treatment structure.

CR Split-Plot		RCB Split-Plot		LS Split-Plot	
Sources	df	Sources	df	Sources	df
				Rows	a-1
		Blocks	r-1	Columns	a-1
A	a-1	A	a-1	A	a-1
Error a	a(r-1)	Error a	(a-1)(r-1)	Error a	(a-1)(a-2)
$Total_W$	ar-1	$Total_W$	ar-1	$Total_W$	a^2 -1
B	b-1	B	b-1	B	b-1
AB	(a-1)(b-1)	AB	(a-1)(b-1)	AB	(a-1)(b-1)
Error b	a(r-1)(b-1)	Error b	a(r-1)(b-1)	Error b	a(r-1)(b-1)
$Total_S$	ar(b-1)	$Total_S$	ar(b-1)	$Total_S$	a^2 (b-1)
TOTAL	abr-1	TOTAL	abr-1	TOTAL	a^2 b-1

7.5 Split-plot designs for three factor factorials.

Split-Split-Plot
1 factor for each split

Sources	df
Blocks	r-1
A	a-1
Error a	(a-1)(r-1)
$Total_W$	ar-1
B	b-1
AB	(a-1)(b-1)
Error b	a(r-1)(b-1)
$Total_S$	ar(b-1)
C	c-1
AC	(a-1)(c-1)
BC	(b-1)(c-1)
ABC	(a-1)(b-1)(c-1)
Error c	ab(r-1)(c-1)
$Total_{SS}$	abr(c-1)
TOTAL	abcr-1

Split-Plot
A & B on whole units, C on sub-units

Sources	df
Blocks	r-1
A	a-1
B	b-1
AB	(a-1)(b-1)
Error a	(ab-1)(r-1)
$Total_W$	abr-1
C	c-1
AC	(a-1)(c-1)
BC	(b-1)(c-1)
ABC	(a-1)(b-1)(c-1)
Error c	ab(r-1)(c-1)
$Total_S$	abr(c-1)
TOTAL	abcr-1

8 AN EXAMPLE OF A SPLIT PLOT

8.1 Consider the following split plot experiment:

- 16 subjects, 8 males and 8 females
- Every subject received each of two exercise treatments (A,B) three days apart
- Within males and females four subjects were randomly assigned to treatment A first followed by B, while the remaining subject received treatment B followed by A
- The response variable was the unresisted ankle simple reaction time
- Name the experimental and treatment design(?).

8.2 The design

A and B are the treatment IDs

	SUBJECT															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Gender	F	F	M	M	F	M	M	M	F	F	M	F	F	F	M	M
Day 1	A	B	B	A	B	A	A	B	A	A	A	B	B	A	B	B
Day 4	B	A	A	B	A	B	B	A	B	B	B	A	A	B	A	A

8.3 The data

A and B are treatments,the data are unresisted ankle simple reaction time

	SUBJECT															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Gender	F	F	M	M	F	M	M	M	F	F	M	F	F	F	M	M
Day 1	A206	B207	B219	A222	B225	A238	A228	B187	A181	A234	A232	B187	B196	A212	B224	B193
Day 4	B204	A202	A237	B228	A259	B226	B194	A201	B187	B249	B204	A206	A223	B195	A227	A219

8.4 The results

Sources of Variation	Fixed/ Random[1]	Ankle Reaction Time df	F/VC	prob
Gender	Fixed	1	0.6	<.5
Subject/Gender	Random	14	236.0	<.01
Day	Random	1	2.0	<.5
Treatment	Fixed	1	10.7	<.01
Gender*Treatment	Fixed	1	0.8	<.4
Residual	Random	13	119.0	

[1] F ratios are reported for tests of fixed effects and variance components for random effects with the probability for the log likelihood chi square test.

Treatment means ± sem for mean ankle reaction time

Treatment	Mean ankle reaction time
A	220±6.2
B	217±6.2

replicates/trt = 16 and sed = 3.9